CORRECTION

Serie 1 : Quality factor of passive components - Chapter 2

Exercise 1:

The series resistive loss R_s of an inductor L_s whose value is equal to 50 nH, is equal to 10 Ω at a frequency f_0 equal to 100 MHz.

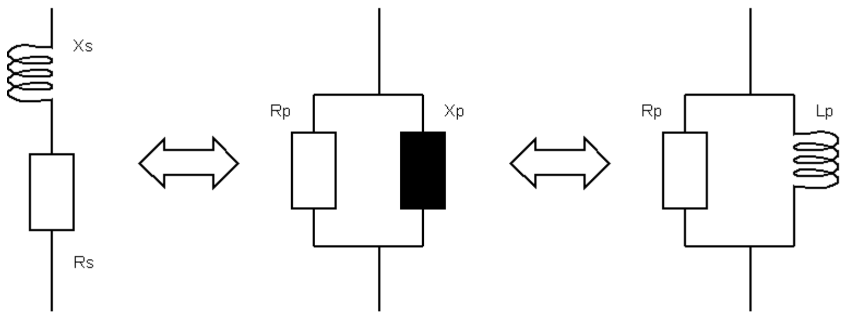
a) Calculate the quality factor Q_s at 100 MHz of the series equivalent circuit.

Answer: $Q_s = \frac{L \cdot \omega}{R_s} = \frac{5 \cdot 10^{-8} \cdot 2\pi \cdot 10^8}{10^1} = 3.14$

b) Transform the series equivalent circuit (L_s, R_s) by the parallel equivalent circuit (L_p, R_p) at 100 MHz by using the transformation from a series equivalent circuit to a parallel equivalent circuit.

Answer:

The equivalent series and shunt circuits are given below, with $X_s = R_s \cdot Q = 10 \cdot 3.14 = 31.4\Omega$:



The equivalent shunt circuit values are given by:

$$R_P = R_S(1 + Q^2) = 10 \cdot (1 + 3.14^2) = 108.7\Omega$$
$$X_P = X_S\left(1 + \frac{1}{Q^2}\right) = 31.4 \cdot \left(1 + \frac{1}{3.14^2}\right) = 34.6\Omega$$
$$L_P = \frac{34.6\Omega}{2\pi \cdot 10^8} = 55.1nH$$

Another way to calculate the equivalent shunt circuit is the following:

Since $Q = \frac{|X_S|}{R_S} = \frac{R_P}{|X_P|}$, then $|X_P| = \frac{R_P}{Q} = \omega \cdot L_P$.

Thus $L_P = \frac{R_P}{\omega \cdot Q} = \frac{108.7}{2\pi \cdot 10^8 \cdot \pi} = 55.1nH$.

c) Calculate the quality factor Q_p at 100 MHz of the parallel equivalent circuit.

Answer:

Since, the working frequency remains the same, we have $Q_P = Q_s = 3.14$.

Exercise 2:

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The internal impedance Rs of the voltage source is equal to 1000 Ohms.
The load impedance R_L of the load impedance is equal to 1000 Ohms.

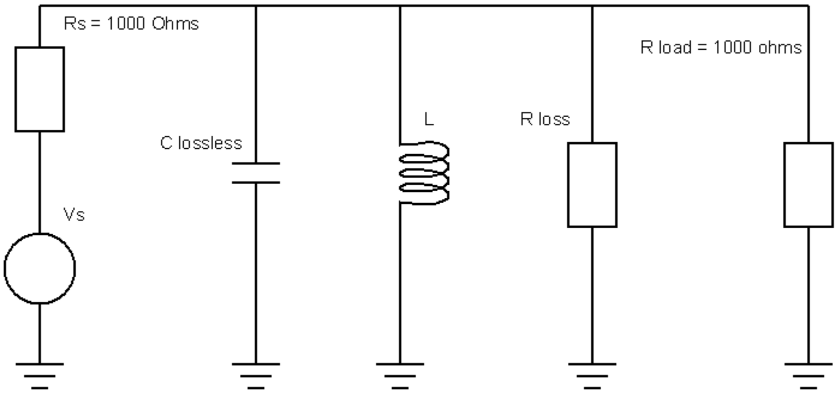
It is assumed that the capacitor of the resonant circuit is lossless and that the quality factor of the inductor of the resonant circuit is equal to 85.

a) Design a parallel resonant circuit to provide a (-3 dB) bandwidth B of 10 MHz at a center frequency f0 equal to 100 MHz.

b) What is the effect of the resistive loss of the inductor on the value of the output voltage V_{out} at the resonance frequency ?

Answer:

a) The corresponding circuit is given below.



Then, we can replace the equivalent Thevenin voltage source constituted by a pure voltage source Vs in series with the resistance Rs by an equivalent Norton current source constituted by a pure current source Is = Vs / Rs in parallel with the resistance Rs.

The specifications of this exercise provide the quality factor of the inductor L when its resistive loss is in series. In the course, we have seen that when we transform the impedance constituted by this inductor L in series with its resistive loss R by another inductor L_p in parallel with a resistive loss R_{losses} then the inductor L_p is equal to L if the quality factor of the inductor is high in front of 1. That is the case in this exercise as it is equal to 85. Therefore, we have that the quality factor of the inductor (without the source and load circuit) is given by:

$$Q_{inductor} = 85 = \frac{R_{losses}}{\omega_0 \cdot L}.$$

We have to determine the value of L and C such that the resonant frequency $f_0 = 100MHz$ and the bandwidth $B_{-3dB} = 10MHz$.

We have the following fundamental formulæ where Q is the Q factor of the whole circuit:

$$f_0 = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{L \cdot C}}$$

$$Q = \frac{R_{total}}{\omega_0 \cdot L} = \frac{R_s \parallel R_{losses} \parallel R_L}{\omega_0 \cdot L}$$

$$R_S = R_L$$

$$Q = \frac{f_o}{B_{-3dB}}$$

$R_{losses} = Q_{inductor} \cdot \omega_0 \cdot L$

Since$R_S = R_L$, we have:

$$Q = \frac{f_o}{B_{-3dB}} = \frac{\frac{R_s}{2} \parallel R_{losses}}{\omega_0 \cdot L} = \frac{\frac{R_s}{2} \parallel Q_{inductor} \cdot \omega_0 \cdot L}{\omega_0 \cdot L} = \frac{\frac{R_s}{2} \cdot Q_{inductor} \cdot \omega_0 \cdot L}{\frac{R_s}{2} + Q_{inductor} \cdot \omega_0 \cdot L} \cdot \frac{1}{\omega_0 \cdot L}$$

Thus, we obtain:

$$Q = \frac{R_s \cdot Q_{inductor}}{R_s + 2Q_{inductor}\omega_0 L}$$

$$\Leftrightarrow Q \cdot [R_s + 2Q_{inductor}\omega_0 L] = R_s \cdot Q_{inductor}$$

$$\Leftrightarrow \omega_0 L = \left[\frac{R_s \cdot Q_{inductor}}{Q} - R_s\right] \cdot \frac{1}{2Q_{inductor}}$$

$$\Leftrightarrow L = \left[\frac{R_s}{2Q\omega_0} - \frac{R_s}{2Q_{inductor}\omega_0}\right]$$

Finally, we have:

$$L = \frac{R_s}{2\omega_0}\left[\frac{1}{Q} - \frac{1}{Q_{inductor}}\right]$$

$$C = \frac{1}{\omega_0^2} \cdot \frac{1}{L}$$

Numeric application:

$R_S = R_L = 1000\Omega$

$\omega_0 = 2\pi \cdot 10^8 \frac{rad}{s}$

$Q = \frac{R_{total}}{\omega_0 \cdot L} = \frac{10^8 Hz}{10^7 Hz} = 10$

$Q_{inductor} = 85$

We have :

$$L = \frac{R_s}{2\omega_0}\left[\frac{1}{Q} - \frac{1}{Q_{inductor}}\right] = \frac{1k\Omega}{2 \cdot 2\pi \cdot 10^8}\left[\frac{1}{10} - \frac{1}{85}\right] = 70.215nH$$

$$C = \frac{1}{(2\pi \cdot 10^8)^2} \cdot \frac{1}{70.215nH} = 36pF$$

$$R_{losses} = Q_{inductor} \cdot \omega_0 L = 85 \cdot (2\pi \cdot 10^8) \cdot (70.215 \cdot 10^{-9}) = 3.75k\Omega$$

b) What is the effect of the resistive loss of the inductor on the value of the output voltage Vout at the resonance frequency ?

Routeq = (Rload . Rlosses) / (Rload + Rlosses)
Vout = Vs Routeq / (Rs + Routeq)

Numerical application:
Rload = 1000 Ohm
Rlosses = 3750 Ohm

Routeq = 789.5 Ohm
Vout = 0.44 Vs

If the quality factor of the inductor would be infinite then the resistance Rlosses would be also infinite and the output voltage Vout would be equal to 0.5 Vs